

# Mathematical Modeling and Analysis of Phenomena

— In Pursuit of Introduction of Science and Engineering —

The essence of science is to derive principles from observations, thereby revealing the simple structures that underlie what appear to be complex phenomena.

Mathematical sciences, in particular, use mathematical approaches to investigate "mathematical models", which are derived through experimentation.

The modern frontier of mathematical sciences considers various new mathematical models, including those for biological and social phenomena, in addition to more traditional models in natural sciences. These models are investigated through analytical approaches as well as numerical simulations in order to understand the phenomena.

The knowledge thus obtained is employed to develop innovative technologies or to predict future developments, and new methodologies are devised for deeper understanding of phenomena.

In its research and educational activities, our course pays particular attention to large-scale and/or highly non-linear complex phenomena.

Particularly in education, we aim at providing students with both scientific perspectives, which help them to understand principles, and engineering perspectives which are useful in solving real problems in manufacturing.

This two-pronged approach gives students comprehensive overviews on both science and engineering in mathematical sciences.



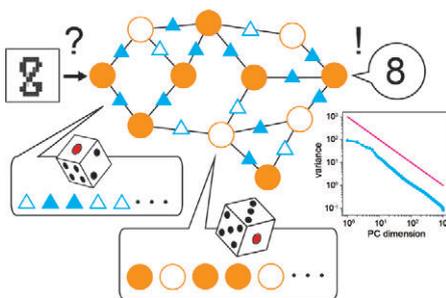
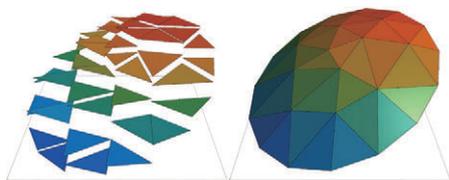
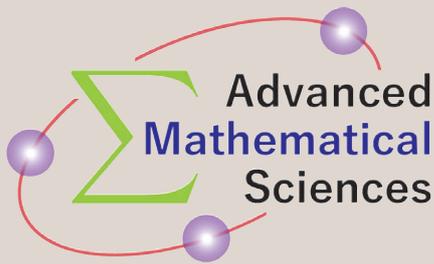
## An Invitation to Mathematical Sciences

In Graduate School of Informatics, we regard "mathematical modeling and analysis of natural, social and biological phenomena" as an important foundation of what we aim at; i.e., "informatics" in the broad sense of the term, which includes not only traditional computer science but also related disciplines such as mathematical sciences, applied physics, etc.

Indeed, we have placed importance on research and education in mathematical sciences since the establishment of our school in 1998 because they are considered to form the academic basis of our disciplines. At the time of inauguration of the school, we considered it appropriate to name ourselves the "Department of Applied Analysis and Complex Dynamical Systems", since "complex dynamical systems" was the phrase that symbolized the advanced mathematical sciences of the time. However, the relentless advancement of mathematical sciences gradually made "complex dynamical systems" not necessarily the best phrase to express what we are. Meanwhile, development of computers and networks has made it possible to deal more easily with so-called "big data," thus making computers and networks increasingly important in social life as well as in research and education in mathematical modeling and analysis of natural, social and biological phenomena. With these developments in mind, we have decided to change the name of our department to "Department of Advanced Mathematical Sciences," as of April 2017. We believe that the new name reaffirms and clarifies our directions for both education and research.

In the broad research area of "advanced mathematical sciences," we are particularly interested in "mathematical modeling and computer simulations of phenomena," which constitute the basis of our activities. Computer simulations and data analyses are now vitally important in several branches of science and engineering. New applications of computational methods are found in biology, social sciences, etc. and, of course, as more conventional applications in mechanics. These new trends in mathematical modeling of phenomena call for new methodologies, including probabilistic and fractal approaches in addition to classical differential equations and discrete models. New developments in computer simulation include large-scale, high-performance computing, use of new computational environments such as multiple-precision arithmetics, etc. Our course conducts research and education in such new and advanced areas of mathematical sciences.

Traditionally in Japan, fundamental education in mathematical sciences has been carried out in schools of science and more application-oriented education has been provided by engineering schools. However, despite its small size, our course has some faculty members with science backgrounds and others with engineering backgrounds. Our aim is to "integrate science and engineering" and develop students with comprehensive perspectives of advanced mathematical sciences. We also emphasize individualized teaching as well as independent learning by each student, in line with Kyoto University's principle of "self-learning based on dialogue."



### Hiroshi Fujiwara

*Professor, Advanced Mathematical Sciences Course*

Professor Fujiwara obtained his B.S. degree from Kyoto University in 1999, and Ph.D. from Kyoto University in Informatics in 2003. His research themes are numerical analysis for partial differential equations, inverse and ill-posed problems, and design and implementation of numerical software.

## Outline

### ■ Laboratory and Teaching Staff

Laboratory	Teaching Staff
<b>Applied Analysis</b>	Jun Kigami/Professor Daisuke Shiraishi/Associate Professor Masayoshi Kubo/Assistant Professor Ryosuke Shimizu/Assistant Professor
<b>Nonlinear Physics (Nonlinear Dynamics and Computational Statistical Physics)</b>	Syuji Miyazaki/Senior Lecturer Kenji Harada/Assistant Professor
<b>Nonlinear Physics (Non-equilibrium Physics and Theoretical Neuroscience)</b>	Toshio Aoyagi/Professor Jun-nosuke Teramae/Associate Professor Hiroki Tutu/Assistant Professor
<b>Computational Mechanics</b>	Hiroshi Fujiwara/Professor Daisuke Kawagoe/Assistant Professor
<b>Industrial Mathematics</b>	Satoshi Taguchi/Professor Tetsuro Tsuji/Associate Professor
<b>Statistical Signal Processing</b>	Kazunori Hayashi/Professor (Secondary appointment : Institute of Liberal Arts and Sciences)

### ■ Curriculum of Advanced Mathematical Sciences Course

Doctoral Program (Informatics)						
<b>3<sup>rd</sup></b>	Doctoral Thesis					
<b>2<sup>nd</sup></b>	Subjects provided by the Course (total 6 credits) Seminar on Mathematical Sciences (Advanced) <i>E</i> (2 credits Mandatory)	<b>Research Guidance</b>				
<b>1<sup>st</sup></b>	Seminar on Applied Analysis (Advanced A & B) <i>E</i> (2 credits each), Seminar on Nonlinear Physics (Advanced A & B) <i>E</i> (2 credits each), Seminar on Applied Mathematical Sciences (Advanced A & B) <i>E</i> (2 credits each)					
Master's Program (Informatics)						
Master's Thesis						
<b>2<sup>nd</sup></b>	Subjects provided by the Course (Optional 8 credits or more) <b>Advanced Subjects (A and B are offered every other year)</b> Topics in Differential Equations A,B (2 credits each), Topics in Nonlinear Analysis A,B (2 credits each), Topics in Applied Analysis I, II (1 credit each), Topics in Nonlinear Dynamics A,B (2 credits each), Topics in Nonequilibrium Physics A,B (2 credits each), Topics in Nonlinear Physics I, II (1 credit each), Topics in Computational Mechanics A,B (2 credits each), Topics in Mathematical Sciences A,B (2 credits each), Topics in Applied Mathematical Sciences I, II (1 credit each)	Seminars and exercises for Master's thesis (Mandatory 8 credits)  Advanced Study in Mathematical Sciences 2 (Assigned to M2, 6 credits)  Advanced Study in Mathematical Sciences 1 (Assigned to M1, 2 credits)				
<b>1<sup>st</sup></b>	<b>Basic Subjects (A and B are offered every other year)</b> Applied Analysis A,B (2 credits each), Nonlinear Physics A,B (2 credits each), Applied Mathematical Sciences A,B (2 credits each)					
General Subjects provided by the School (Compulsory elective 2 or more credits and 4 credits or less for ◎ subjects)		<b>Specific subjects provided by the school</b>				
<b>Interdisciplinary Subjects of the Perspectives in Informatics</b> ◎Perspectives in Informatics 1 ◎Perspectives in Informatics 2 ◎Perspectives in Informatics 3 ◎Perspectives in Informatics 4 ◎Perspectives in Informatics 5 (2 credits each)						
◎Perspectives in Platform Studies (2 credits), Computational Science, Introduction (2 credits), Computational Science, Exercise A (1 credits), ◎Information and Intellectual Property (2 credits), Innovation and Information (2 credits), Information Analysis and Management (2 credits), Information Analysis and Management, Exercise (1 credits), Social Contributions through Informatics (1 credits), Internship in the Field of Informatics (1 credits),						
Prior to admission	Calculus	Linear Algebra	Fundamentals of Ordinary Differential Equations	Fundamentals of Complex Analysis	Mechanics of Particles, Systems of Particles, and Rigid Bodies	Academic Foundations at Undergraduate Level for Study at the Course

Note: Subjects marked with the letter "E" will be provided in English.

## Admission, Curriculum and Other Efforts in Our Course

In study and research of advanced mathematical sciences, one needs both basic mathematical skills as well as knowledge of his/her specialist field. Accordingly, in the entrance exam, all applicants for our Master's course are required to solve basic problems in linear algebra and calculus together with one problem of their choice related to their respective specialties. Final decisions on acceptance are made through interviews with those applicants who score above a certain level in the written exam, since it is not desirable to make such decisions based only on the results of the written exam where one point may be the difference between a pass mark and failure. The interviews will be conducted by all the faculty members in order to determine if applicant's interests match the expertise of our faculties. In the entrance exam for the Ph.D. course, final decisions on acceptance are made based on the achievements of each candidate in his/her research work, which are evaluated in interviews conducted by all the faculty members.

Our curriculum for the Master course consists of both general and specialized subjects. All students are advised to take three general subjects, which help them to develop both scientific and engineering perspectives in mathematical sciences. Research

advice is given mainly on a one-to-one basis, taking into account each student's aptitude. For those wishing to go on to Ph.D. courses, we provide Seminar II (for second-year students), which is designed to give students opportunities to learn advanced topics in addition to receiving standard research mentoring. In the Ph.D. course, students can receive mentoring not only from their advisers but also from other professors in our course and affiliated professors from science and engineering schools. This system gives students access to a broader spectrum of state-of-the-art knowledge in the mathematical sciences. Students thus have opportunities to deepen their expertise and to obtain a broad appreciation of mathematical sciences from both scientific and engineering perspectives, which we believe to be a unique feature of the education provided by our course.

In addition, almost every year, we offer open, public seminars on topical subjects.



## Applied Analysis Laboratory

### *Analysis in the 21st Century*

#### ■ Fractal Analysis, Fractal Geometry

I am interested in the mathematical theory of problems concerning heat and wave propagation in fractal concept models –new models for the natural world. [Jun Kigami]

#### ■ Structure of Brownian Motion and Random Walk

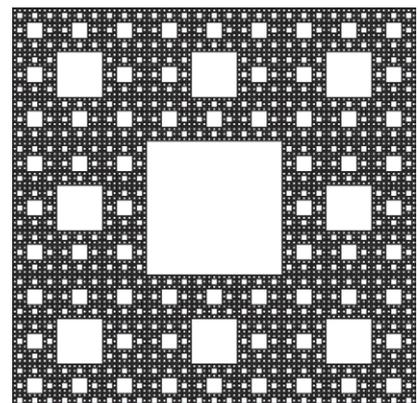
How does the trace of Brownian motion look like? What can we say about the structure of random walk trace? Such questions have fascinated probabilists and mathematical physicists for a long time, and they continue to be an unending source of challenging problems. I am interested in the nature of sample paths of these fundamental processes. [Daisuke Shiraishi]

#### ■ Inverse Problem Analysis, Numerical Analysis of (Partial) Differential Equations, Partial Differential Equations, Brain Model Mathematical Research

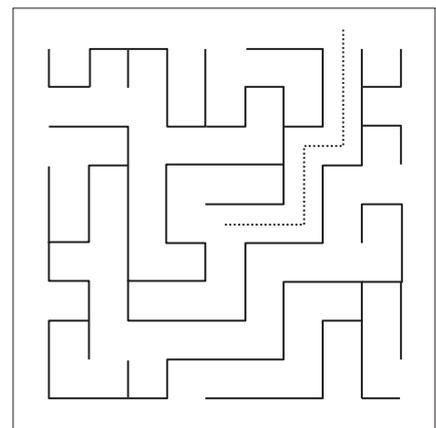
I mathematically analyze partial differential equations that appear in mathematical physics and mathematically and numerically analyze the inverse problems found in these partial differential equations, where the unknown coefficients of these inverse problems are determined by observed data. [Masayoshi Kubo]

#### ■ Analysis and geometry of fractals and metric spaces

Aiming to address problems that have been identified in the study of metric space geometry, I research basic theories in analysis such as nonlinear potential theory on complex spaces including fractals. Specifically, I aim to shed light on the relationship with quasiconformal geometry and on phenomena unique to fractals that cannot occur with a classic approach. [Ryosuke Shimizu]



Sierpiński carpet



Uniform spanning tree (solid line) and dual path (dotted line)

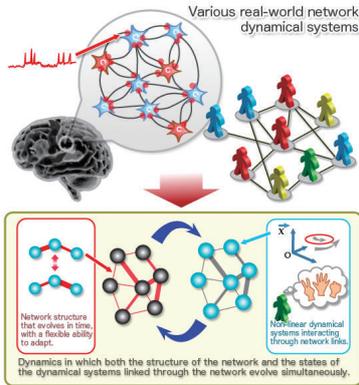
**Outline**

**Nonlinear Physics Laboratory**

**Group of Non-equilibrium Physics and Theoretical Neuroscience**

**From nonlinear physics to theory of networks, as well as living and neural systems**

I am particularly interested in systems composed of many simple elements that, through cooperative interactions, come to exhibit complex behavior and high-level functions, such as not only many physical systems, but also biological and social systems. Co-evolution involving the intricate interplay between the dynamics of the network and the elements is a key concept for understanding the self-organized, flexible nature of real-world network systems. I study such cooperative phenomena in systems of this kind, focusing on rhythmic phenomena and chaos from the perspective of nonlinear dynamics. [Toshio Aoyagi]

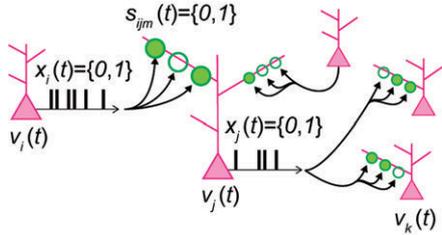


Neural systems and social networks are nonlinear dynamic systems that have a high degree of freedom and develop as their dynamic elements change the structure of their interconnections.

**Nonlinear physics of computation and learning in the brain**

The brain is a highly complex network composed of about 100 billion of neurons. Spike propagation along the network and plasticity of synaptic connection of them cause high-dimensional nonlinear dynamics, which is the nature of neural computation and learning. Central questions, however, still remain elusive.

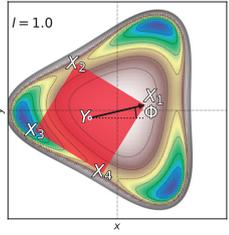
What is intelligence? What is underlying principles of neural computation and learning? Our recent study, for example, reveals significant roles of spontaneous fluctuation in neural computation. By integrating neuroscience, computer science, and nonlinear physics, here we are trying to answer these questions and trying to develop fully brain-inspired AIs. [Jun-nosuke Teramae]



Conceptual diagram of neurons (pink) in the brain interacting through synapses (green circles) that transmit impulse currents in a process called "spike firing." The process can be mathematically analyzed by modeling the states of neurons and synapses as random variables.

**Mathematical models for molecular machines**

Biological molecular motors are amazing machines that generate useful movement (as a pump or a porter). The relationship between the structures of proteins and their highly efficient energy conversion, even under a fluctuating environment, remains mysterious. I explore the underlying principles of such molecular machines with mathematical models. [Hiroki Tutu]

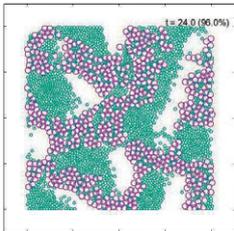


Rotary ratchet model consisting of a rotor (rigid body consisting of four points) and a rotor housing (potential) that simulates a Wankel rotary engine

**Group of Nonlinear Dynamics and Computational Statistical Physics**

**Study on powders and charged particles under cyclic external forces**

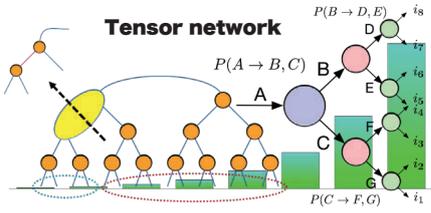
Inspired by experiments in which several micron-size charged corpuscles are injected into an AC trap, a simplified version of a Paul ion trap, and prior studies into the particle model of ion channels and its anomalous transport, we mainly study collective motions of granular materials and charged corpuscles, which are subject to periodic external force. We also propose mathematical models for numerical analysis that faithfully reproduce experiment settings and analyze critical phenomena incidental to collective motions of macroscopic charged corpuscles injected into an AC trap and various types of bifurcations that occur in vibrating powder. [Syuji Miyazaki]



It is known that when two different powders, one of large and one of small grain size, are horizontally vibrated, the two powders separate, causing a stripe pattern to form. We discovered that when powders, one of large and one of small grain size, each one with a size distribution centered around its average grain size, are vibrated horizontally, the two powders separate, causing a mesh pattern to form.

**Statistical physics using computational approaches**

Using the computational approach, we advance statistical physical research for the emerging properties in complex systems consisting of many elements with nonlinear interactions. My research areas range from the atomic scale to the real world, e.g., quantum critical phenomena and infectious diseases. Based on tensor network formalism, I am also working on informational processing technology, such as classical and quantum machine learning. [Kenji Harada]



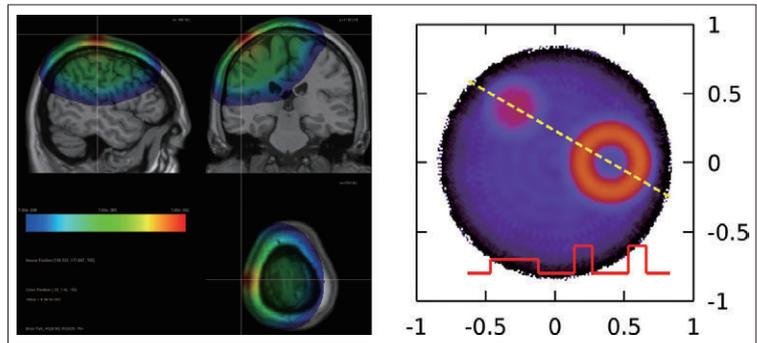
Researchers actively study new compression methods for large data sets and deep learning models with exponentially growing parameters using tensor network representation.

## Computational Mechanics Laboratory

*Computer simulations: development of highly reliable numerical analysis methods and their applications to inverse problems*

The understanding of phenomena with mathematical models is applied to a wide range of scientific and technological advancements. This lab analyzes inverse problems in topics such as the development of numerical computation methods and medical imaging to understand and apply phenomena described in partial differential equations and integral equations. Specifically, we specialize in analysis of numerical computation methods, multiple-precision computation, massively parallel computation, and numerical analysis to shed light on the properties of mathematical models.

[Hiroshi Fujiwara, Daisuke Kawagoe]



Simulation of light propagation in an organism (left) and example of identification of optical properties (right)

## Industrial Mathematics Laboratory

*Toward new fluid mechanics for non-equilibrium flows*

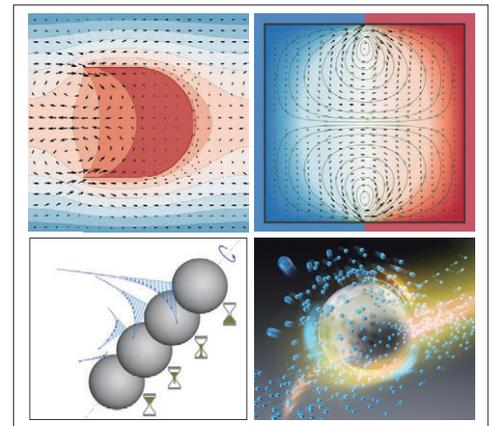
In our group we investigate the behavior of non-equilibrium flows based on kinetic theory describing the collective behavior of innumerable particles.

We aim at understanding mechanical and/or thermodynamic properties of non-equilibrium flows both theoretically and numerically. We also aim at elaborating continuum theory and applying it to non-equilibrium flows, by deriving suitable mathematical models for non-equilibrium flows.

[Satoshi Taguchi]

Recent advances in micro/nanoscale technologies require the understanding of transport phenomena in micro/nanoscale and their control. For this reason there is a growing interest in the research of non-equilibrium flows. In particular, we aim at investigating moving boundary problems for non-equilibrium flows and the motion of tiny materials (or particles) driven by non-equilibrium effects from the view point of mathematical sciences and from experimental view points. We also aim at integrating them in industrial applications.

[Tetsuro Tsuji]



Unidirectional flow around a heated U-shaped body (top left)  
Flow caused by a discontinuous wall temperature in a vessel (top right)  
Flow caused by a rotating sphere impulsively set into motion (bottom left)  
The Magnus effect of a spinning microparticle (bottom right)

## Statistical Signal Processing Laboratory

*Integration of Model-based and Data-driven Approaches*

In the discipline of statistical signal processing, we try to systematize methodologies for extracting useful information from raw observation data. This has a wide range of applications in telecommunications, as well as measurement, image processing, and biological signal processing. Statistical signal processing is a valuable tool for tackling various kinds of problems relating to data science, including data sensing, collection, transmission, analysis, and utilization. We have recently been studying methods for solving underdetermined linear inverse problems that make use of the sparsity and discreteness of signals.

[Kazunori Hayashi]

Example of optical communication receiver configuration using "deep unfolding"  
Combining traditional telecommunications knowledge with a data-driven approach makes it possible to determine all parameters in an optical communication receiver system at once based on the loss function at output.

