

An English Translation:

Mathematics for Dynamical Systems

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Let $a, b, c, d \in \mathbb{R}$ be constants and consider the differential equation

$$t^2 \frac{dx}{dt} + (at + b)x = ct + d, \quad (1)$$

where $b \neq 0$. For a positive integer n , a solution is called an n th-order polynomial solution if it is an n th-order polynomial of t containing a nonzero n th-order term. Answer the following questions.

- (i) Obtain a necessary and sufficient condition for equation (1) to have a first-order polynomial solution, and express the condition with a, b, c and d .
- (ii) Let $n > 1$ be an integer. Obtain a necessary and sufficient condition for equation (1) to have an n th-order polynomial solution, and express the condition with a, b, c, d and n .
- (iii) Obtain a necessary and sufficient condition for equation (1) to have no n th-order polynomial solution for any positive integer n , and express the condition with a, b, c and d .